...ERSITY :: KURNOOL

...COND III SEMESTER B.Sc EXAMINATIONS FEB/MAR.2023

SUBJECT :MATHEMATICS

PAPER CODE & TITLE : C3310 ABSTRACT ALGEBRA

Time: 3 Hours

Max Marks: 75 Marks

## SECTION-A

**Answer any Five Questions.**

**(5 x 5M = 25M)**

1. In a group $(G,.)$, $for\ a, b \in G$,Show that $(a.b)^{-1} = b^{-1}a^{-1}$.
2. Find order of each element of the group of fourth roots of unity w.r.to multiplication.
3. Give an example to show that union of two subgroups is not a subgroup.
4. Define a normal subgroup of a group and give two examples.
5. Show that a group homomorphism $f: G \to G^1$ is one-one if and only if $\ker f = \{e\}$.
6. Determine whether the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix}$ is even or odd?
7. Prove that every cyclic group is abelian.
8. Prove that cancellation laws hold good in an integral domain.

## SECTION-B

**Answer ALL the Five Questions.**

**(5 x 10M = 50 M)**

9. (a) Show that the set of integers $Z$ form an abelian group with respective to the operation $*$defined as $a * b = a + b + 2, \forall\, a, b \in Z$. **(OR)**

   (b) Prove that the order of an element of a group is at most the order of the group.

10. (a) Prove that anon-empty complex $H$ of a group $G$ is a subgroup of G if and only if $a, b \in H \Rightarrow a.b^{-1} \in H$ **(OR)**

    (b) Prove that order of a subgroup of a finite group always divides order of the group.

11. (a) Prove that a subgroup $H$ of a group $G$ is a normal subgroup of G if and only if every left coset of $H$ in $G$ is also a right coset of $H$ in $G$. **(OR)**

    (b) If $f: G \to G^1$ is an epimorphism then prove that $G^1$ is isomorphic to some quotient group of $G$.

12. (a) State and prove Cayley's theorem. **(OR)**

    (b) Prove that every group of prime order is cyclic. Is the converse true? Justify your answer with an example

13. (a) Prove that $\mathbb{Q}\sqrt{2} = \{a + b\sqrt{2}: a, b \in \mathbb{Q}\}$ is a field. **(OR)**

    (b) Prove that every finite integral domain is a field.

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